Performance' Improvement on Target Date Fund using GARCH Volatility Forecasting Model

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Abstract: The depletion problem of the national pension plan is emerging as life expectancy increases and the fertility rate decreases. The retirement pension system is being introduced in earnest to supplement the national pension system. The Target Date Fund, introduced to prepare for retirement, rebalances its portfolio through Glide Path, which has a fixed ratio of risky assets according to the subscriber's life cycle. The purpose of this study was to propose a new Glide Path that simultaneously considers the life cycle and stock market volatility, and to analyze the possibility of improving the performance of the TDF portfolio through empirical analysis. To this end, we first predict stock market volatility for determining investment risk and derive a Glide Path reflecting the predicted volatility. Stock market volatility, which has the greatest influence on the new Glide Path, is predicted using the GARCH model. If the volatility is expected to increase, the TDF risk will be managed by reducing the risky assets. Results of the study using financial market data from 1987 to 2021 showed as follows. First, the asymmetric phenomenon of volatility was significant, and the usefulness of the asymmetric GARCH model was revealed. Second, the proposed Glide Path was able to lower the risk of TDF funds by lowering the risky asset incorporation ratio in the stock market crash periods such as 1998, 2008, and 2020. Third, the TDF portfolio applied to the proposed model showed higher returns and lower standard deviation, improving Sharpe Ratio. Fourth, the model proposed in the long-term investment performance showed a lower maximum draw down than the comparative model. It was revealed that the TDF performance could be improved by reflecting the market risk.

Keywords: Glide Path, Glosten-Jagannathan-Runkle Generalized Autoregressive Heteroskedasticity Model, Market Volatility, Sharpe Ratio, Target Date Fund

1. Introduction

As the average life expectancy increases and the interest rate decreases, the income replacement ratio of the national pension plan, the main source of retirement income for the Korean people, is decreasing. Accordingly, the retirement pension system was introduced as an income source that could supplement the national pension plan. The Target Date Fund (TDF) is a representative retirement pension product, and the fund manager adjusts the proportion of risky and safe assets according to the target date of the fund subscriber. In general, the Glide Path, the asset allocation strategy for TDF funds, is set to lower the ratio of risky assets such as stocks as the subscriber's retirement time approaches. The domestic TDF market has surpassed 10 trillion won as of 2022, showing rapid growth[1]. However, the low-interest-rate paradigm lowers the TDF returns, resulting in problems with retirees' future income.

The investment performance of a fund is affected by the ratio of risky assets to be invested or the selection of investment stocks. TDF's Glide Path for subscribers who have more than 30 years left in retirement starts at around 80%, and is designed to gradually reduce the ratio of risky assets as retirement

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approaches, maintaining only 20% to 30% at retirement. These asset allocation criteria cannot reflect the risk of market fluctuations because the risky asset incorporation ratio is fixed considering only the life cycle of the fund subscriber[2]. In fact, in the face of the 2008 global financial crisis, the TDF, which was subscribed to by the prospective retiree in 2010, suffered heavy losses, which severely damaged retirement funds for the prospective retiree[3]. Basu et al. (2011) pointed out the problem of fixed glide path (FGP) with a predetermined risky asset incorporation ratio, proposed a new glide path that dynamically adjusts the ratio of risky assets to the target return of TDF funds arising from stock price fluctuations[4].

Could it be possible to improve the investment performance of the TDF, which is important as a retirement fund for prospective retirees? The purpose of this study is to propose a new variable Glide Path (VGP) that adjusts the risky asset incorporation ratio of TDF funds while simultaneously considering not only the life cycle of TDF fund subscribers but also the market risk, and analyze whether investment performance can be improved through empirical analysis of TDF funds using the proposed Glide Path. The proposed Glide Path is an asset allocation strategy that hedging the stock market risk by lowering the predetermined risky asset incorporation ratio for maximizing the TDF profitability when the market volatility is predicted downward. The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model, which is known to well reflect the characteristics of time-varying risks observed in the actual stock market, is used[5]. In line with this, the study was conducted to propose a new Glide Path to improve the performance of TDF funds using 35-year long-term actual financial market data.

2. Theoretical Background

2.1 Target Date Fund

TDF is a representative financial product for retirement that invests under the Glide Path, which regulates the investment ratio of risky assets such as stocks and safe assets such as bonds according to the subscriber's life cycle with the aim of raising retirement money for fund subscribers. If the expected retirement time is 2040, the goal of raising retirement funds can be achieved by joining funds such as TDF 2040. As retirement approaches, TDF also reduces the proportion of investment in risky assets, such as stocks.

Until now, glide path studies have mainly focused on fixed glide path studies that use only the subscriber's life cycle as a variable. Most studies argue the advantage of the right-downward glide path, which reduces the ratio of risky asset incorporation over the life cycle[6]. Conversely, there are studies that argue for the usefulness of the right-upward glide path[7][8].

Since the fixed glide path does not reflect market risk factors, it pointed out the problems caused by large fluctuations in the market, and showed that dynamic fund management compared TDF target returns and realized returns resulted in 80% better results than the performance of the existing fixed glide path[4]. Yoon (2010) established an appropriate risk budget for each subscriber's life cycle and empirically analyzed the dynamic glide path reflecting market risks at that time in the US stock market, showing that large losses can be avoided in the global financial crisis as in 2008[3]. Kang et al. (2019) showed that TDF's annual return improved through a mixed strategy of increasing the investment in developed markets when the economy is in a slump, and expanding the proportion of investment in emerging markets when the economy recovers[9]. Kim (2021) proposed a dynamic Glide Path considering retirement time and predictive volatility and showed that the volatility index released by the Korea Exchange is highly useful[10]. Moon et al. (2021) used the transaction volume of the ETF (Exchange-Traded Fund) market to calculate market expectations and propose a glide path that reflects

them[11].

The domestic TDF market has grown rapidly, surpassing KRW 10 trillion in size, and is expected to increase to KRW 50 trillion by 2030[12]. In order to improve the profitability of pension assets, competition among asset managers who operate TDF funds has been intensifying recently with the introduction of default options. Asset managers began to develop their own new Glide Paths and use them in practice for differentiation strategies. At this point, the study on the new Glide Path proposed by this study has great implications.

2.2 GARCH Models

In general, when the stock market falls, the standard deviation of stock returns tends to increase[13]. Therefore, the standard deviation of stock returns is useful as a risk indicator for the stock market. In fact, the inverse relationship between volatility and direction is observed, that is, the market volatility rises when stock prices fall and falls when stock prices rise[14]. In particular, there is also an asymmetry in which the increase and decrease in market volatility[14]. The GARCH model reflects these characteristics well[14].

Kim (2010) predicted the volatility of the KOSPI 200 stock index through various symmetrical and asymmetric GARCH models in the domestic stock market, and showed that the asymmetric GARCH model had excellent performance[15]. Lamar et al. (2015) predicted international cotton prices using the statistical models ARIMA (Autoregressive Integrated Moving Average) and the GARCH model, showing that the asymmetric GARCH model outperformed the comparative models[16]. Lin (2018) predicted China's stock index volatility and found that the predictive performance of the asymmetric GARCH model was superior to that of the comparative model[17]. Most of the research results on volatility support the asymmetric GARCH models that volatility moves asymmetrically.

Recently, research on a hybrid prediction model that combines the GARCH model and the machine learning model is also active. Kim and Choi (2017) showed that the volatility prediction results of the GARCH model and the Support Vector Machines (SVM) hybrid model were excellent[13]. Lu et al. (2022) compared the GARCH model and the LSTM-GARCH model in a volatility prediction study of the Indian stock market, and showed that the GARCH model had better predictive power than the LSTM-GARCH model[18].

3. Research Methodology

3.1 Research Design

3.1.1 GARCH Volatility Prediction Model

The volatility observed in the stock market shows clustering and asymmetry. Attempts to model the observed stylized volatility were introduced by the Nobel Prize-winner Engle (1982) as an ARCH (Autoregressive Conditional Heteroskedasticity) model[19]. Since then, the ARCH model has developed into a generalized ARCH (GARCH) model, and its usefulness has been proven in empirical analysis as it has developed into a model that well explains the volatility clustering and volatility asymmetry observed in the actual market. A representative GARCH model was proposed by Glosten, Jagananthan, and Runke (1993) as the GJR-GARCH model, which is used to estimate the market volatility and predict future volatility in this study[20].

Equations (1) and (2) show the GJR-GARCH model. Equation (1) is the estimation equation of the daily return, and Equation (2) is the estimation equation of the volatility of the next day of the KOSPI stock index return[20].

σ

$$\boldsymbol{r}_{t} = \boldsymbol{\mu} + \boldsymbol{\varepsilon}_{t}, \quad \boldsymbol{\varepsilon}_{t} \sim \boldsymbol{N}(\boldsymbol{\mu}, \boldsymbol{\sigma}_{t}^{2}) \tag{1}$$

$$^{2}_{t+1} = \omega + \alpha \varepsilon_{t}^{2} + \beta \sigma_{t}^{2} + \gamma \varepsilon_{t}^{2} I_{t}, \qquad (2)$$

where
$$I_t = 1$$
 if $\varepsilon_t < 0$ and 0 otherwise.

It is assumed that the stock return follows a normal distribution with an average μ , and volatility is measured by ARCH term α , GARCH term β , and I_t are dummy variables with 1 when the residual ε_t is negative, and γ is the degree of volatility asymmetry. The last dummy variable increases volatility on the day when stock prices fall below average, and the increase is also greater than the decrease in volatility on the day when stock prices rise. Therefore, Equation (2) reflects the clustering phenomenon and asymmetry of volatility actually observed in the stock market[13].

In this study, the GJR-GARCH model of Equation (2) was estimated using past stock market data, and the market volatility of the next day was predicted using the estimation equation. Historical data for estimating the model were used for 5 years, and daily volatility over the next 5 years was predicted in an iterative manner using the estimated GARCH model. Specifically, equations (1) and (2) were estimated using the KOSPI stock index return data from 1987 to 1991, and then daily forecast volatility from 1992 to 1996 was calculated. Next, equations (1) and (2) were estimated using the KOSPI stock index return data from 1987 to 1991, to 2001 was calculated. Repeat and finally, equations (1) and (2) were estimated using the KOSPI stock index from 2012 to 2016, and then daily forecast volatility from 2017 to 2021 was calculated. Using a total of six estimates in five-year units, the volatility of the KOSPI stock index for 30 years from 1992 to 2021 was predicted. In order to calculate the annualized volatility, the square root of the estimated value in Equation (2) is taken and then multiplied by the square root of 252 corresponding to one year's business days to calculate the final volatility, which is the standard deviation of the stock return.

3.1.2 Variable Glide Path and TDF Design

The Variable Glide Path proposed in this study seeks to improve profitability by lowering the risky asset incorporation ratio by judging the stock market's volatility as a risk signal based on the fixed glide path that only considers the age of TDF fund subscribers. [Fig. 1] is a typical fixed glide path example.



[Fig. 1] Fixed Glide Path Example

For people who have more than 30 years left until retirement, the ratio of risky assets will start at around 80%, and if retirement left 25 years, the ratio of risky assets will be lowered to 70%. Over time, the ratio will continue to be reduced. When the retirement time approaches, the ratio of risky assets will be reduced to 20%. Based on the fixed glide path in Figure 1, if the predicted volatility of the proposed GJR-GARCH model is expected to increase or decrease above a certain level, the risky asset ratio at the time is reduced or increased by a certain ratio to minimize risk and maximize profitability. In this study, if the predicted volatility rises more than a% from the previous low, the risky asset ratio is reduced by b%, and if it falls more than a% from the previous high, the ratio is increased by b%.

In this study, it was assumed that TDF consists of a portfolio of risky assets represented by the KOSPI stock index and risk-free assets represented by RF. Therefore, if volatility is expected to rise by more than a%, the portfolio will be rebalanced by reducing the investment ratio in the KOSPI stock index by b% and increasing the ratio of RF by b%.

3.2 Analytical Data

In this study, data on the daily KOSPI stock index (KOSPI) and the daily interest rate of 364-day monetary stabilization securities (RF) were used to simulate the TDF portfolio performance for the domestic stock market. RF is special distribution securities issued by the Bank of Korea for the purpose of controlling the amount of currency in the market, and unlike stocks and securities with high investment risk, they are the safest investment targets. The data collection period is a total of 9,194 days, which is 35 years from January 5, 1987 to December 30, 2021. The data were obtained from the data guide[21]. [Fig. 2] shows the trend of KOSPI and RF in the data analysis period.



[Fig. 2] Long-term Trends on KOSPI and Interest Rate

The KOSPI stock index rose from 264.82 points on January 5, 1987 to 2,977.65 points on December 30, 2021. It can be seen that the interest rate continued to decline during the data analysis period. The lowest value of the KOSPI stock index was 264.82 points on January 5, 1987, and the highest value was 3,305.21 points on July 6, 2021. The interest rate was found to be at least 0.592% to up to 19.21%.

The daily return on data is calculated as shown in Equation (3).

$$r_t = \frac{p_t - p_{t-1}}{p_{t-1}} \times 100(\%)$$
(3)

where r_t is the return on day t, p_t is the price on day t.

[Table 1] summarizes the basic statistics on the KOSPI and RF for the data period.

Statistics	KOSPI	RF	
Average	0.038	-0.021	
Standard Deviation	1.535	0.917	
Min	-12.019	-9.474	
Max	11.946	12.691	

[Table 1] Basic Statistics on Daily Returns(%)

During the data analysis period, the daily return on the KOSPI stock index averaged 0.038%, the standard deviation was 1.535%, the minimum value was -12.019%, and the maximum value was 11.946%. The daily return on interest rate averaged -0.021%, continuing to decline during the analysis period, with the standard deviation of 0.917%, the minimum value of -9.474%, and the maximum value of 12.691%.

4. Results and Discussions

4.1 GARCH Volatility Prediction Result

The GJR-GARCH model of Equations (1) and (2) was estimated by dividing the 5-year period for the total analysis period of 35 years. The estimation of the model used the maximum likelihood estimation[22].

[Table 2] shows the estimation results of the GJR-GARCH model in the data period.

Period	ω	α	β	γ
1987~1991	0.1968	0.1340	0.6833	0.1605
1992~1996	0.0596	0.0605	0.8501	0.0993
1997~2001	0.0300	0.0487	0.9310	0.0406
2002~2006	0.0541	0.0000	0.8993	0.1422
2007~2011	0.0542	0.0003	0.8879	0.1651
2012~2016	0.0255	0.0000	0.9038	0.1087

[Table 2] GJR-GARCH Model Estimations

The estimated results were not different from most of the existing research results, except for the part where the estimated coefficient of the GARCH term was estimated to be low in the 1987-1991 period. In the variance equations for all sub-periods, $\alpha + \beta < 1$ implies that KOSPI index has finite variance[23]. The γ value measuring the degree of asymmetry in volatility is between 0.0406 and 0.1651, showing statistical significance at the 1% significance level. It can be seen that there is an asymmetry of volatility.

We predicted future volatility using the estimated results of the GJR-GARCH model. Specifically, the results estimated from 1987 to 1991 were applied to Equation (2) to predict the daily volatility from 1992 to 1996. By repeatedly applying, the predicted volatility was calculated from the final period, 2017-2021, and the daily volatility prediction value for a total of 30 years between 1992 and 2021 was obtained.

4.2 TDF Portfolio Performance

For the suggested Gide Path, we apply 50% and 20% for a and b, and the resulting Variable Glide Path is shown in [Fig. 3].



[Fig. 3] Variable Glide Path based on Predicted Volatility

Assuming that asset allocation has been conducted in the actual financial market for 30 years using the variable glide path of subscribers retiring in 2021, 30 years after joining the TDF in 1992, the investment results of the TDF fund as shown in [Table 3] can be obtained.

Performance	Variable Glide Path	Fixed Glide Path	
Daily Average Return	0.0330%	0.0259%	
Standard Deviation	0.8548%	0.9924%	
Sharpe Ratio	0.0386	0.0261	
Maximum Draw Down	-51.58%	-78.71%	

[Table 3] Performance on TDF

In [Table 3], TDF performance using the Variable Glide Path, which reflected the volatility prediction results proposed in this study, showed a higher daily average return than the fixed Glide Path, which considered only the age of the prospective retiree. On the contrary, the standard deviation for evaluating the risk of investment results was lower in the proposed model. Sharpe Ratio, an indicator of investment performance that measures the size of returns versus investment risk, showed a higher for the proposed model, showing better performance in TDF investment than traditional Glide Path. This result was consistent with Yoon(2010) who showed better performance of TDF in 2008 global financial crisis using a dynamic asset allocation strategy that considers the fund risk budget[3].

From the perspective of subscribers, the proposed model was lower in the Maximum Draw Down, which measures the drop from the time of the maximum return to the minimum return during the TDF subscription period. As such, it showed that from the subscriber's point of view, the stress from the decline in the fund's return during the subscription period was less. Rikey and Yan(2022) documented that mutual funds with low Maximum Draw Down will generate excess return in the future, and the result is magnified when stock market are in turbulent states[24].

[Fig. 4] shows the equity curve of TDF over the entire 30 years.



[Fig. 4] Equity Curve on TDF(1992~2021)

In [Fig. 4], the results of the investment of the proposed model showed that the decline in profits in 1998, 2008, and 2020 is smaller than that of the benchmark model. Since predictive volatility increases as stock prices fall, it can be seen that the risky asset ratio has been lowered and adjusted according to the proposed model. Risk management was added by reflecting market volatility, and as a result, the maximum draw down was lowered, resulting in positive performance.

5. Conclusions

This study proposed a new Glide Path that adjusts the ratio of TDF portfolio to risk assets while simultaneously reflecting the age of subscribers and the risk of the stock market, and empirically analyzed using domestic financial market data for 35 years. As life expectancy increases and the birth rate decreases, the importance of TDF is increasing. The results of the empirical analysis were as follows. First, the estimation results of the GJR-GARCH model considering the clustering and asymmetry of market volatility showed that the asymmetry of volatility was significant. Second, GlidePath, which considered predictive volatility, flexibly adjusted the ratio of incorporating risky assets according to the risk situation of the market. Third, TDF's investment performance based on the proposed model showed higher profitability and lower risk, and the performance evaluation index Sharpe Ratio showed higher results. Fourth, the proposed model was also low for the stress received from the subscriber's point of view. It was revealed that the TDF returns could be improved through the suggested Glide Path that reflects not only the age of prospective retirees but also market risks.

This study has some limitations. First, it is necessary to increase the predictive performance of stock market volatility through expansion to deep learning models. Second, it is assumed that a TDF portfolio consisting only of domestic stocks and bonds is assumed. Despite these limitations, our study is of academic and practical significance in that it proposes a new glide path that replaces the traditional glide path and reveals the usefulness of the variable glide path. In future studies, it is necessary to improve investment performance through diversification of portfolios to overseas stocks, bonds, and alternative investments by expanding investment assets.

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References

[1] http://www.kfr.co.kr/nsr/nsr100000Detail?NewsroomId=4132, Jan 26 (2022)

- [2] H. G. Kang, K. H. Bae, S. T. Yang, C. H. Choi, An investment strategy based on life and business cycles, Korean Journal of Financial Studies, (2019), Vol.48, No.6, pp.721-754. DOI: http://dx.doi.org/10.26845/KJFS.2019.12.48.6.721
- [3] Y. Yoon, Glide path and dynamic asset allocation of target date funds, Journal of Asset Management, (2010), Vol.11, pp.346-360.
 DOI: http://dx.doi.org/10.1057/jam.2010.20
- [4] A. K. Basu, A. Byrne, M. E. Drew, Dynamic lifecycle strategies for target dateretirement funds, Journal of Portfolio Management, (2011), Vol.37, No.2, pp.83-96. DOI: http://dx.doi.org/10.3905/jpm.2011.37.2.083
- [5] R. K. Yelamanchili, Stock market returns, data frequency, time horizon, return distribution density and GARCH models, IUP Journal of Applied Economics, (2021), Vol.20, No.1, pp.29-46.
- [6] D. M. Blanchett, Revisiting the optimal distribution glide path, Journal of Financial Planning, (2015), Vol.28, No.2, pp.52-61.
- [7] W. Pfau, M. Kitces, Reducing retirement risk with a rising equity glide path, Journal of Financial Planning, (2013), Vol.27, No.1, pp.38-45.
 DOI: http://dx.doi.org/10.2139/ssrn.2324930
- [8] L. Delorme, Confirming the value of rising equity glide paths: Evidence from a utility model, Journal of Financial Planning, (2015), Vol.28, No.5, pp.46-52.
- [9] H. G. Kang, K. H. Bae, S. T. Yang, C. H. Choi, An investment strategy based on life and business cycles, Korea Journal of Financial Studies, (2019), Vol.48, No.6, pp.721-754. DOI: http://dx.doi.org/10.26845/KJFS.2019.12.48.6.721
- [10] S. W. Kim, Dynamic glide path using retirement target date and forecast volatility, Journal of Convergence for Information Technology, (2021), Vol.11, No.2, pp.82-89. DOI: http://dx.doi.org/10.22156/CS4SMB.2021.11.02.082
- [11] M. D. Moon, S. W. Kim, H. S. Choi, A study on dynamic glide path of Target Date Fund reflecting market expectations, Knowledge Management Research, (2021), Vo.22, No.3, pp.17-29. DOI: http://dx.doi.org/10.15813/kmr.2021.22.3.002
- [12] http://news.mt.co.kr/mtview.php?no=2022110913571654544, Nov 11 (2022)
- [13] S. W. Kim, H. S. Choi, Estimation of GARCH models and performance analysis of volatility trading system using support vector regression, Journal of Intelligence and Information Systmes, (2017), Vol.23, No.2, pp.107-122.
- [14] S. W. Kim, Negative asymmetric relationship between VKOSPI and KOSPI 200, Journal of the Korean Data Analysis Society, (2010), Vol.12, No.4, pp.1761-1773.
- [15] S. W. Kim, A study on developing a VKOSPI forecasting model via GARCH class models for intelligent volatility trading systems, Journal of Intelligence and Information Systems, (2010), Vo.16, No.2, pp.19-32.
- [16] A. Lama, G. K. Jha, R. K. Paul, B. Gurung, Modelling and forecasting of price volatility: An application of GARCH and EGARCH models, Agricultural Economics Research Review, (2015), Vol.28, No.1, pp.73-82.

DOI: http://dx.doi.org/10.5958/0974-0279.2015.00005.1

- [17] Z. Lin, Modelling and forecasting the stock market volatility of SSE composite index using GARCH models, Future Generation Computer Systems, (2018), Vol.79, No.3, pp.960-972.
 DOI: http://dx.doi.org/10.1016/j.future.2017.08.033
- [18] V. Mahajan, S. Thakan, A. Malik, Modeling and forecasting the volatility of NIFTY 50 using GARCH and RNN models, Economies, (2022), Vol.10, No.5, pp.1-20. DOI: https://doi.org/10.3390/economies10050102
- [19] R. F. Engle, Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation, Econometrica, (1982), Vol.50, No.4, pp.987-1007.
 DOI: https://doi.org/10.2307/1912773
- [20] L. Glosten, R. Jagannathan, D. Runke, Relationship between the expected value and the volatility of the nominal excess return on stocks, Journal of Finance, (1993), Vol.48, pp.1779-1801. DOI: http://dx.doi.org/10.1111/j.1540-6261.1993.tb05128.x
- [21] http://www.dataguide.co.kr, Oct 30 (2022)
- [22] C. Francq, J. M. Zakoian, Maximum likelihood estimation of pure GARCH and ARIMA-GARCH processes, Bernoulli, (2004), Vol.10, No.4, pp.605-637.
- [23] Y. J. Zhang, Y. Fan, H. T. Tsai, Y. M. Wei, Spillover effect of US dollar exchange rate on oil price, Journal of Policy Modeling, (2008), Vol.30, No.6, pp.973-991. DOI: http://dx.doi.org/10.1016/j.jpolmod.2008.02.002
- [24] T. B. Riley, Q. Yan, Maximum drawdown as predictor of mutual fund performance and flows, Financial Analysts Journal, (2022), Vol.78, No.4, pp.59-76.